Optimal change detection under internal fluctuations in precision

Shaiyan Keshvari, Ronald van den Berg, Wei Ji Ma

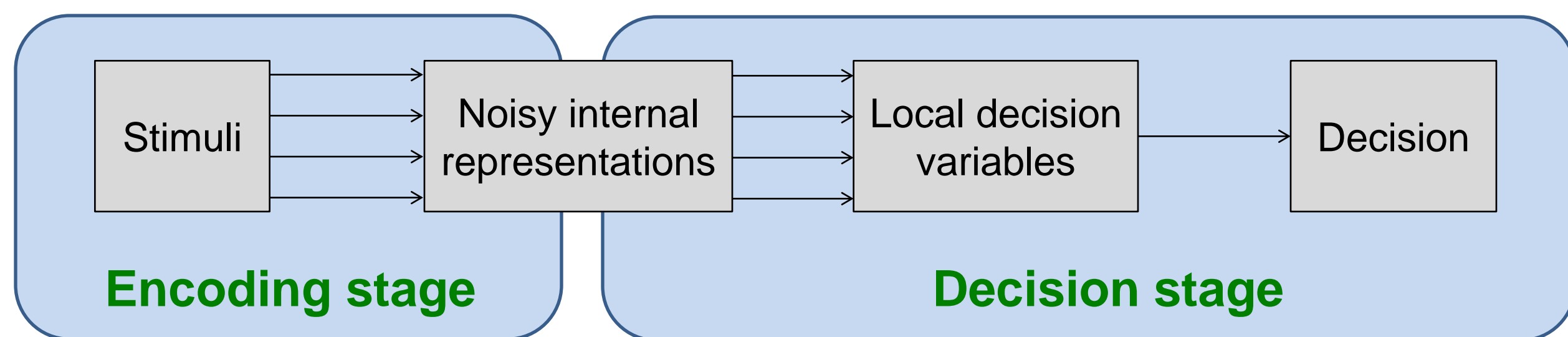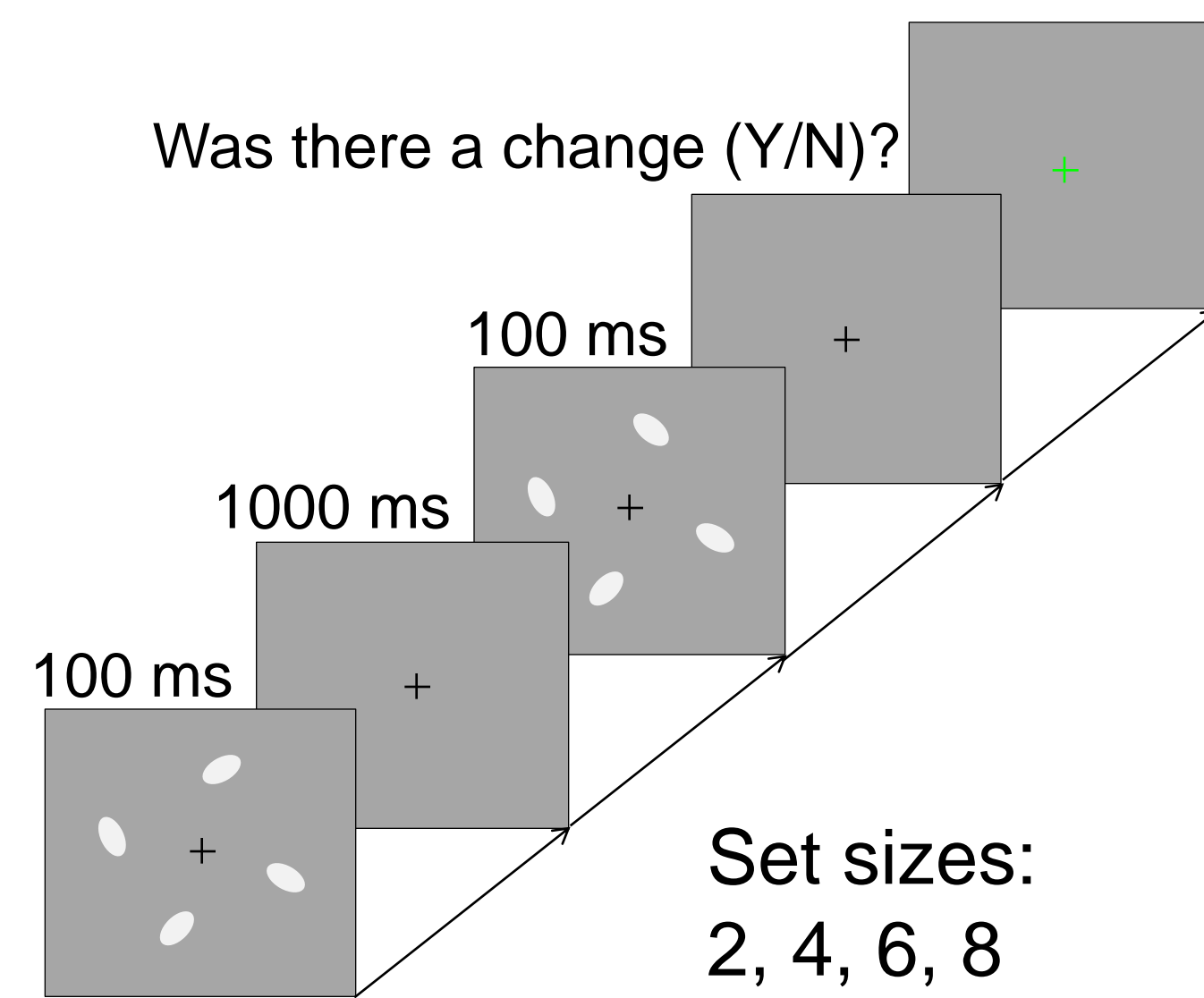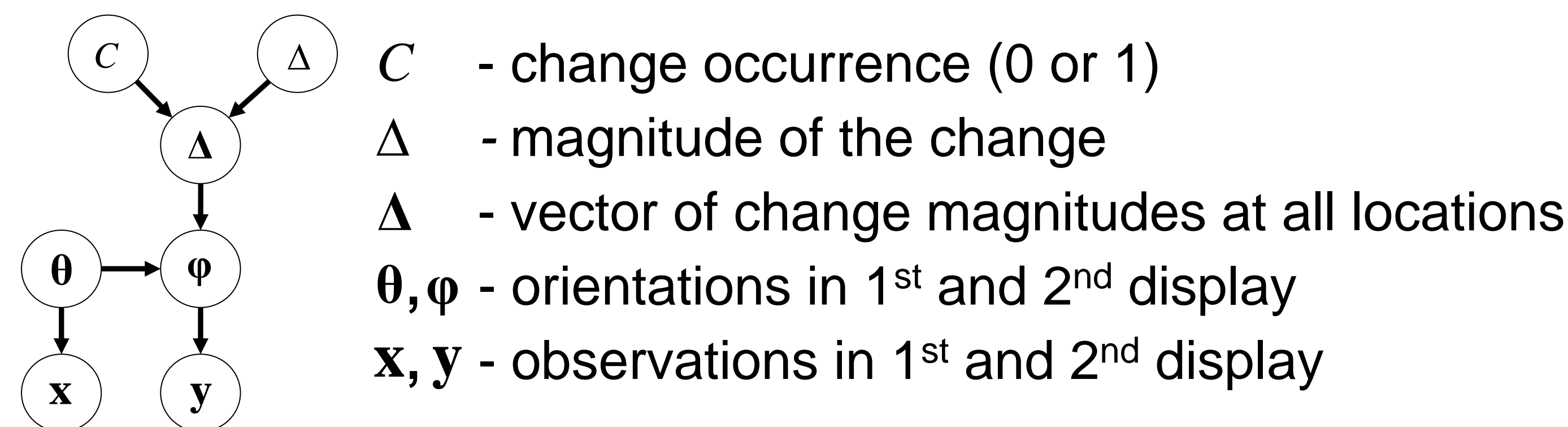Baylor College of Medicine, Department of Neuroscience, Houston TX, USA

Experiment 1

- Change detection is a popular paradigm in studies of visual short-term memory (VSTM).

- We model this task using a combination of a VSTM **encoding** stage and an optimal-observer model for the **decision** stage.

Was there a change (Y/N)?

100 ms

1000 ms

100 ms

Set sizes: 2, 4, 6, 8

Encoding stage		Decision stage	
Stimuli	Noisy internal representations	Local decision variables	Decision

Modeling the encoding stage

Observations are modeled as noisy (Wilken & Ma, 2004) with a Von Mises distribution (circular Gaussian).

$$p(x \mid \theta; \kappa) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(x-\theta)}$$

The precision with which a stimulus is encoded is determined by the concentration parameter, κ, and depends on which model of VSTM is assumed.

Models of VSTM

$N=2$ $N=5$

Item-limit (IL)
(Pashler, 1988)

Slots-plus-averaging (SA)
(Zhang and Luck, 2008)

Fixed precision (FP)
(Palmer, 1990)

NEW Variable precision (VP)

In the **variable-precision** model, resource is drawn (independently for each item) from a Gamma distribution. Here, the mean of this distribution is inversely proportional to set size.

Fluctuations in precision might represent fluctuations in attention.

Probability — Encoding precision
$N=2$, $N=4$, $N=6$, $N=8$

Modeling the decision stage

C - change occurrence (0 or 1)

Δ - magnitude of the change

$\boldsymbol{\Delta}$ - vector of change magnitudes at all locations

$\boldsymbol{\theta}, \boldsymbol{\varphi}$ - orientations in 1st and 2nd display

$\mathbf{x}, \mathbf{y}$ - observations in 1st and 2nd display

The optimal observer infers C from $\mathbf{x}$ and $\mathbf{y}$, by integrating out nuisance variables

$$d = \frac{p(C=1 \mid \mathbf{x}, \mathbf{y})}{p(C=0 \mid \mathbf{x}, \mathbf{y})} = \frac{p(\mathbf{x}, \mathbf{y} \mid C=1)}{p(\mathbf{x}, \mathbf{y} \mid C=0)} \frac{p(C=1)}{p(C=0)} = \cdots$$

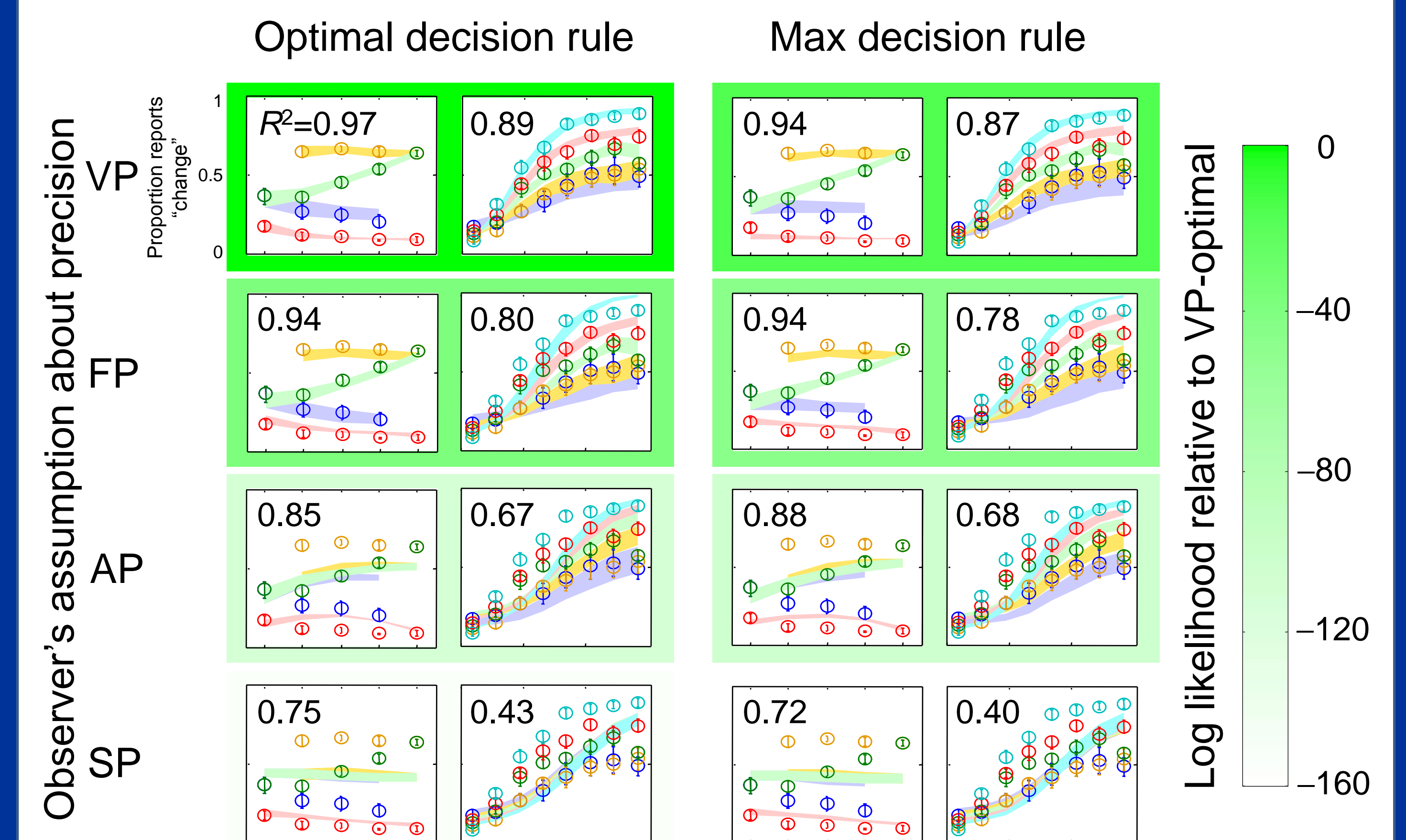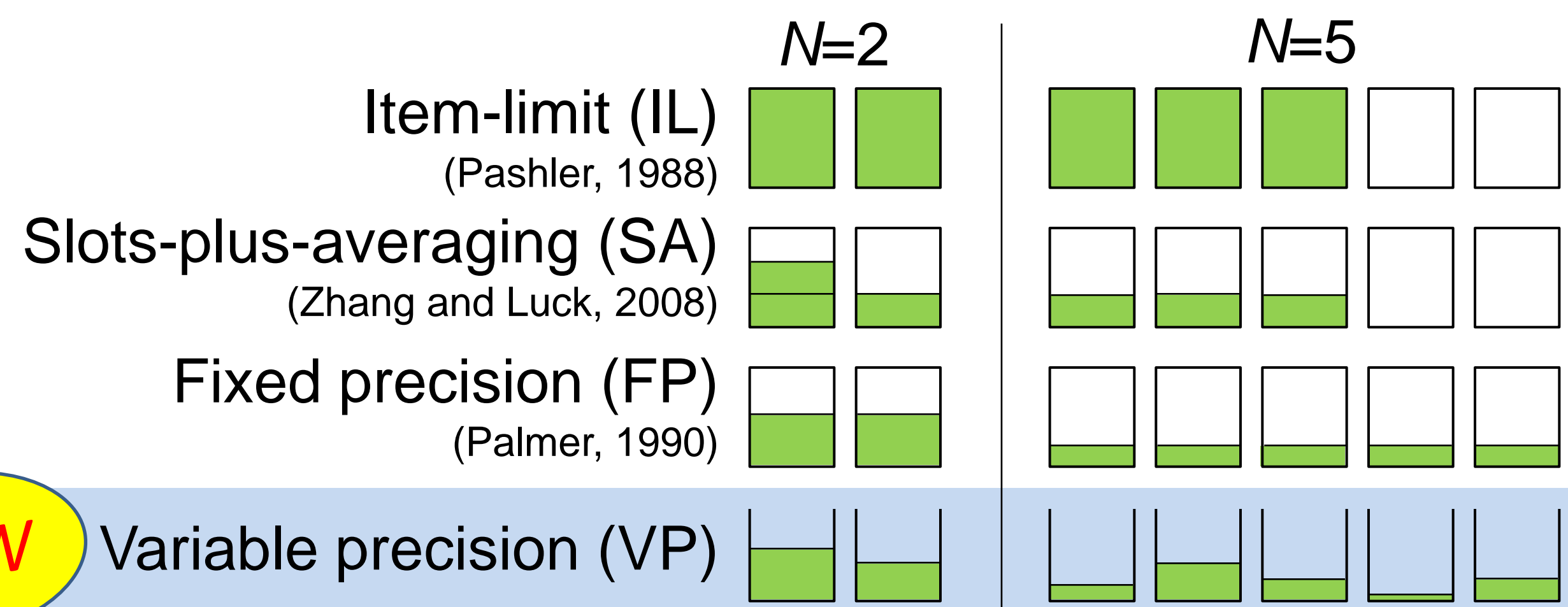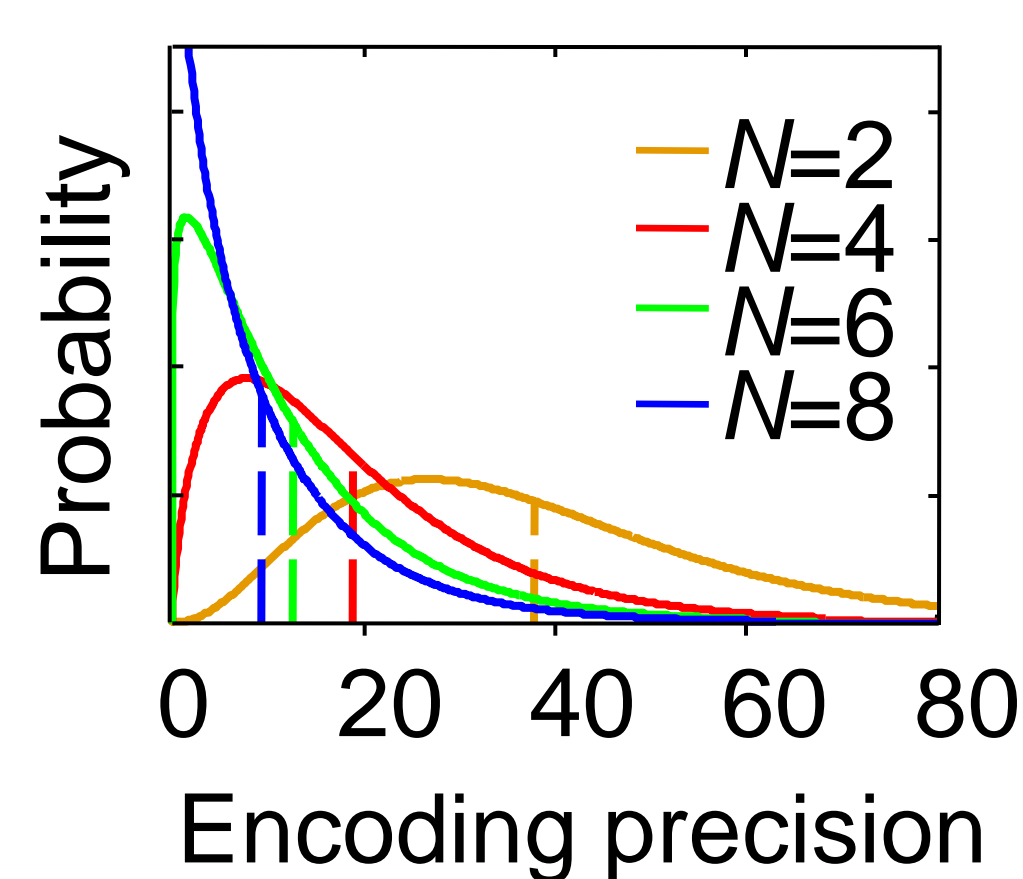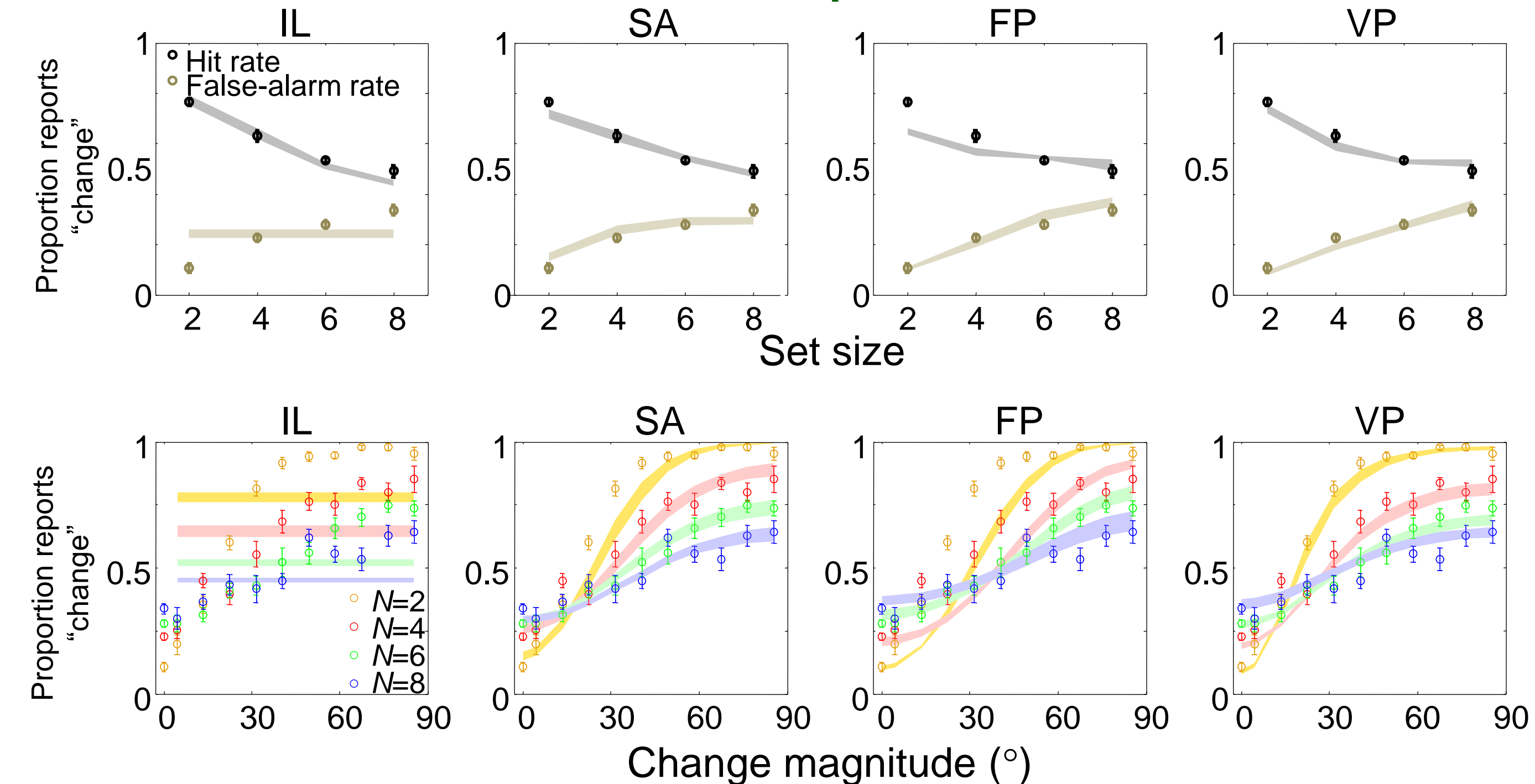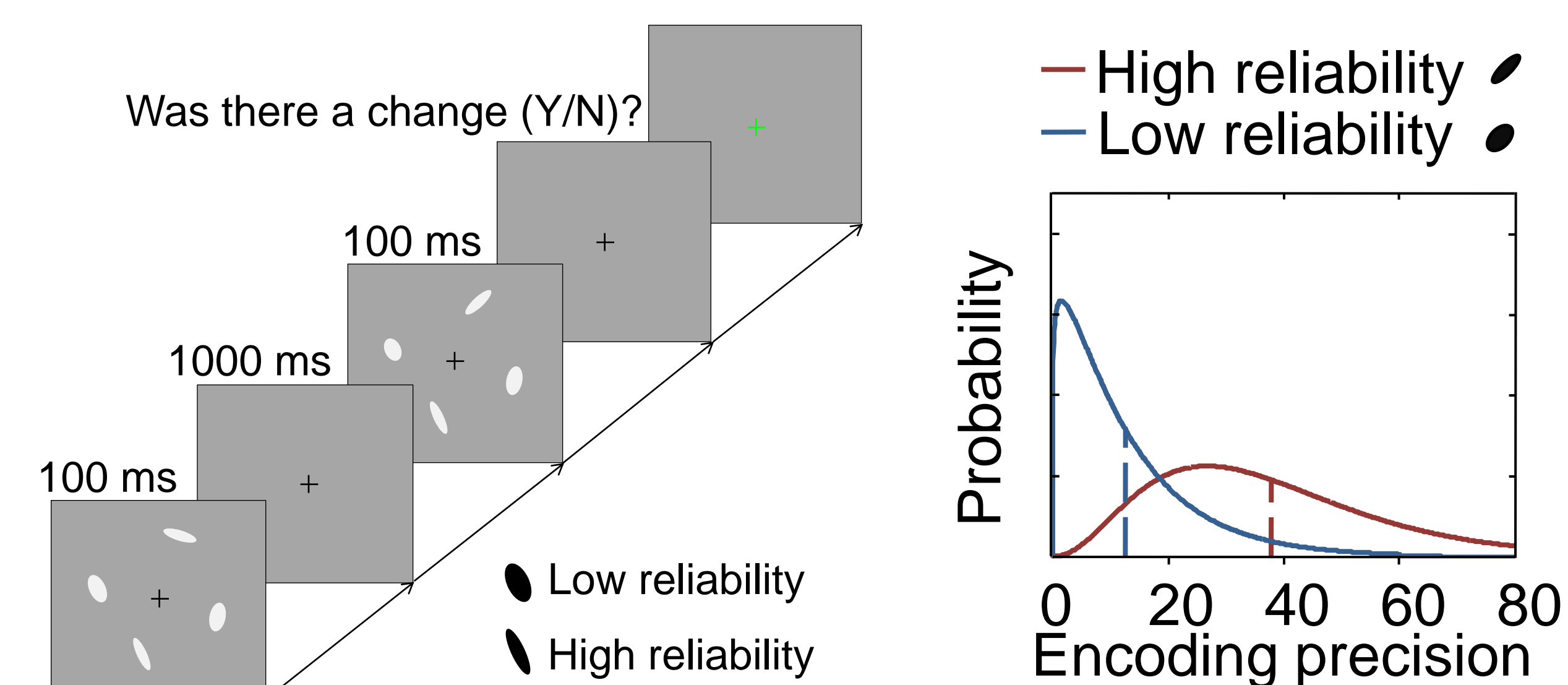$$= \frac{1}{N} \frac{p_{change}}{1-p_{change}} \sum_{i=1}^{N} \frac{I_0(\kappa_{x,i}) I_0(\kappa_{y,i})}{I_0\left(\sqrt{\kappa_{x,i}^2 + \kappa_{y,i}^2 + 2\kappa_{x,i}\kappa_{y,i}\cos(y_i - x_i)}\right)}$$

Results of Experiment 1

IL   SA   FP   VP

Hit rate
False-alarm rate

Proportion reports "change" — Set size

Proportion reports "change" — Change magnitude (°)

$N=2$, $N=4$, $N=6$, $N=8$

Bayesian model comparison results

Orientation   Color

IL   SA   FP   IL   SA   FP

Log likelihood relative to VP

Main findings
1) VP is most likely model for encoding stage
2) Optimal decision rule fits very well

Experiment 2

Do human observers optimally take into account both internally and externally induced variability in stimulus reliability?

Was there a change (Y/N)?

100 ms

1000 ms

100 ms

High reliability
Low reliability

Probability — Encoding precision

Low reliability
High reliability

Results of Experiment 2

In the decision stage, the observer may not incorporate complete knowledge of item-to-item encoding precision. We therefore consider the following **observer assumptions**:

- Variable precision   (correct)
- Fixed precision   (κ_{LOW} and κ_{HIGH} are constants)
- Average precision   ($\kappa_{LOW} = \kappa_{HIGH}$ = average κ)
- Single precision   ($\kappa_{LOW} = \kappa_{HIGH}$ = constant)

We consider the optimal and a suboptimal (max) decision rule.

Optimal decision rule   Max decision rule

Observer's assumption about precision

VP $R^2 = 0.97$ 0.89   0.94   0.87
FP   0.94   0.80   0.94   0.78
AP   0.85   0.67   0.88   0.68
SP   0.75   0.43   0.72   0.40

Number of high-reliability stimuli, N_H Magnitude of change (°)

Log likelihood relative to VP-optimal

Hits: low-reliability change
Hits: high-reliability change
Hits combined
False alarms

$N_H=0$, $N_H=1$, $N_H=2$, $N_H=3$, $N_H=4$

Bayesian model comparison

Optimal   Max

FP   AP   SP   VP   FP   AP   SP

Log likelihood relative to VP-optimal

Main finding
The most likely model is the one in which precision is variable, the observer has complete knowledge about precision, and uses the optimal decision rule.

Conclusions

- Visual short-term memory is limited by a continuous and variable resource.

- Human observers perform optimal inference in change detection, taking into account both internally and externally induced fluctuations in encoding precision.

Palmer, J. (1990). Attentional limits on the perception and memory of visual information.
Pashler, H. (1988). Familiarity and visual change detection.
Wilken, P. & Ma, W. J. (2004). A detection theory account of change detection.
Zhang, W. & Luck, S. J. (2008). Discrete fixed-resolution representations in visual working memory.